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ELECTROMAGNETIC ATTENUATION PROPERTIES OF CLAY AND
GRAVEL SOILS

Dennis L. Brown

Air Force Weapons Laboratory
Kirtland Air Force Base, New Mexico

July 1975

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Dennis L. Brown, Capt, USAF

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Final Report for Period February 1974 - June 1974

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This final report was prepared by the Air Force Weapons Laboratory under Job Order 21044A16. Major George D. Ballentine (DEZ) was the Laboratory Project Officer-in-Charge.

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This technical report has been reviewed and is approved for publication.

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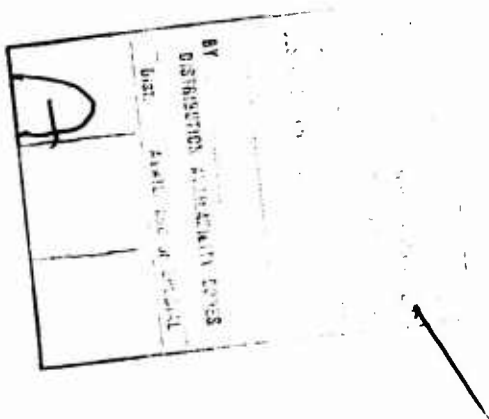
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <p>The electromagnetic attenuation properties of clay and gravel soils were measured as a function of moisture content and frequency. Measurements were done at frequencies in the 0.5 to 4.5 GHz range. Soil samples were compounded in the laboratory at approximately 10, 50, and 90 percent saturation. Sample thickness was in the range of 2.5 to 20.3 cm. Each homogeneous sample was sealed in a polyethylene container to retain the total moisture and maintain a constant moisture profile with depth. The results indicate that this</p> <p style="text-align: right;">(over)</p>								

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ABSTRACT (cont'd)

method cannot satisfactorily determine the thickness of gravel or clay soil layers over a range of moisture contents which would be experienced in field conditions.

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PREFACE

The experimentation required for this study was performed by Mr. Doyle A. Ellerbruch, National Bureau of Standards in Boulder, Colorado with the assistance of Professors Hon-yim Ko and Edward Sampson, Department of Civil and Environmental Engineering, University of Colorado. Statistical analysis of the data was performed by Major George D. Ballentine, Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico.

CONTENTS

<u>Section</u>		<u>Page</u>
I	INTRODUCTION	5
II	EXPERIMENTAL PROCEDURE	6
III	EXPERIMENTAL RESULTS	13
IV	CONCLUSIONS	17
	APPENDIX: THEORETICAL MODEL	18
	ABBREVIATIONS AND SYMBOLS	23
	REFERENCES	24

SECTION I

INTRODUCTION

The Air Force must periodically evaluate airfield pavements to establish their operational capabilities and program any required maintenance. A new method of nondestructively evaluating the condition and load carrying capacity of airfield pavements is currently under development, but one shortcoming of that method is that layer thickness information is required to evaluate the pavement. At the present time there is no method available to nondestructively determine the thickness of the various layers of a pavement system, thus necessitating the use of "as-built" drawings or digging a test pit.

This study was undertaken to determine the feasibility of using active microwave techniques to differentiate between and accurately measure the thickness of various layers in a pavement system. Some previous work has been done to measure the thickness of Portland Cement Concrete (PCC) and Asphaltic Cement Concrete (ACC) materials with relatively good results, but very little is known about the applicability of this technique to construction materials below the PCC or ACC surface course.

When using active microwave techniques to characterize these natural construction materials three factors are of prime importance: soil type, moisture content, and the frequency of the microwave signal. The first step in demonstrating the ability to measure the thickness of these material layers is to show the ability of the microwave signal to penetrate the material under varied moisture contents. The measure of this ability to penetrate used in this report is the skin depth or e-folding distance, which is the depth at which the wave has been attenuated to $1/e$ of its original value. (The theoretical model which is the basis of this work is included as an appendix.) If the skin depth is sufficient to indicate that the wave can penetrate the full depth of the pavement system (which varies from 61 to 122 cm), and that for similar materials under similar conditions the measured skin depth is the same, the feasibility of using this technique to measure pavement thickness would be established.

SECTION II

EXPERIMENTAL PROCEDURE

The testing in this study involved measuring the skin depth in two different soils (gravel and clay), at three different saturation levels in each (10, 50, and 90 percent), at microwave frequencies in the range of 0.5 to 4.5 GHz.

The gravel was blended by mixing concrete aggregates in calculated proportions, so that the grain-size distribution curve as shown in figure 1 was obtained. Soil so prepared classifies as a poorly graded gravel (GP) in the Unified Soil Classification system (see ref. 4). Compaction characteristics were obtained by the modified Proctor technique and are given in figure 2. From figure 2, a moisture content was selected for making soil for the test bed in order to achieve a compacted degree of saturation of approximately 10 percent. The actual degree of saturation of the test soil (based on volume calculations) could not be accurately predicted until the bed had been prepared.

Two boxes of identical dimensions (1.33 m x 1.22 m x 20.3 cm) were built for making two beds of soil simultaneously. Slight variation resulted in the density of the soil bed prepared. The boxes were lined with polyethylene plastic sheets to prevent leaking of soil moisture.

Mixing of the dry gravel with the water was carried out in a concrete mixer, and compaction of the bed was accomplished by means of a gasoline-powered vibratory machine. After compaction, the box was weighed by supporting it on a knife edge on one side and a proving ring on the opposite side. Thus, with the wet density, moisture content and grain specific gravity known for the soil bed, the void ratio, dry density, and the degree of saturation could be calculated.

The above techniques were employed in making the soil beds at 10 percent degree of saturation. After measurements had been made on them, the degree of saturation of the soil was brought up to 50 percent by introducing water from the surface in an amount calculated from the dry density of the prepared soil. The box was again weighed to provide data for accurate determination of new wet density, and therefore, degree of saturation. Thus, by this method, compacted dry density of the soil stayed constant from one saturation state of the test specimen to another. The same technique was also used to bring saturation to

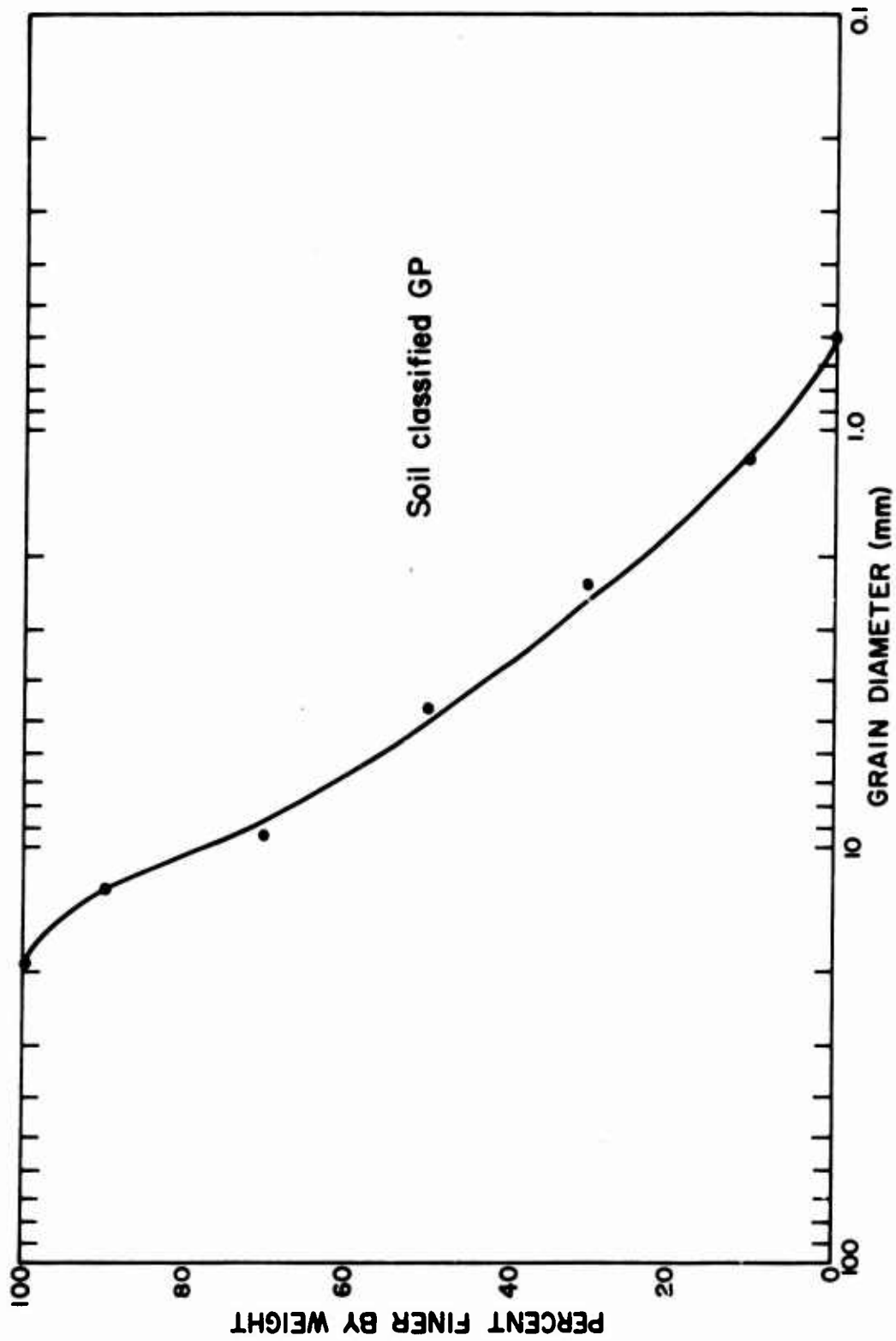


Figure 1. Grain Size Distribution of Gravel Samples

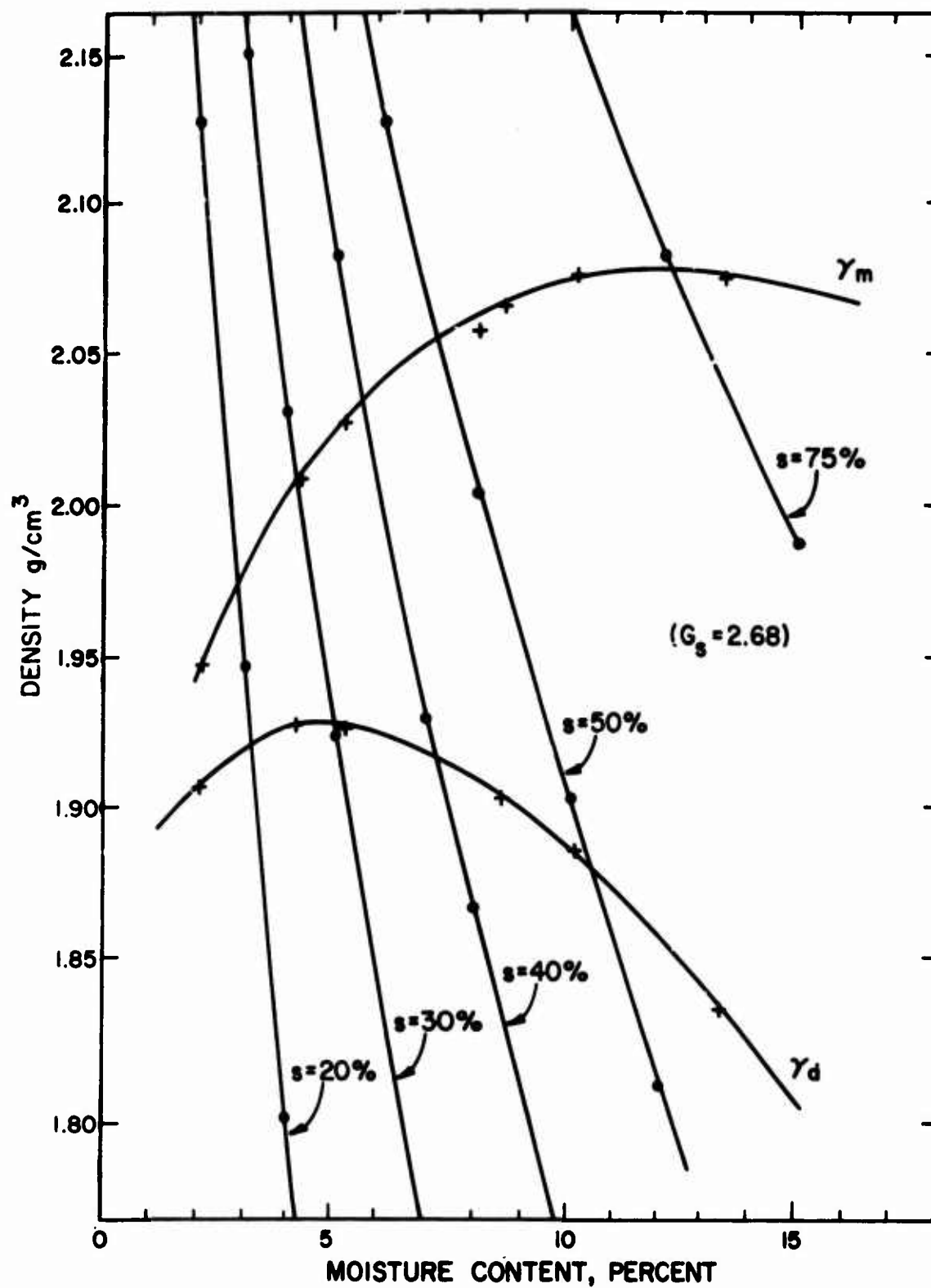


Figure 2. Modified Proctor Compaction of Gravel

90 percent, although the presence of trapped air in the soil presented some difficulties in achieving that high degree of saturation.

Data on the two soil beds at each of the three saturations are given in table 1. Target values for moisture content were 10, 50, and 90 percent. Table 1 gives the actual values for each sample.

Some difficulties were encountered in locating a soil with a CL classification because of the large quantities required. Finally, a soil was selected which had to be dried, crushed, and sieved, retaining the minus number 50 sieve portion. Soil used in the experiments has the grain-size distribution curve as shown in figure 3, which also gives its Atterberg limits. Thus, it classifies as a CL in the United Soil Classification System. Compaction characteristics are shown in figure 4.

Dry soil was mixed with an appropriate amount of water in a soil mixer in batches of 110 kg. After placing the soil in the box, it was necessary to compact it with a hand compactor to achieve the desired density and, therefore, the required degree of saturation. After measurements were taken of this soil bed, the soil was remixed with additional water to bring it up to the moisture content necessary to achieve 50 percent saturation. This time, however, a gasoline-powered, rammer-type compaction machine was used to attain the higher density. For the 90 percent saturation, the soil bed with 50 percent saturation was percolated with water through four plumbing fittings on the bottom of the box. This process of increasing the saturation, aimed at maintaining a reasonable compaction of the soil (that of the 50 percent saturated sample), and avoiding the need to compact a clay soil at 90 percent saturation, took about four days to accomplish and worked satisfactorily. Data for the clay beds are shown in table 1.

Table 1
PROPERTIES OF SOIL BEDS

Soil Bed	Moisture Content (percent)	Dry Density (g/cm ³)	Degree of Saturation (percent)
Gravel (Bed 1)	2.0	1.802	11.4
	8.6	1.802	48.9
	15.1	1.802	85.9
Gravel (Bed 2)	2.0	1.874	12.8
	7.4	1.874	47.3
	12.7	1.874	81.2
Clay	3.5	1.352	9.6
	11.0	1.528	39.3
	23.2	1.528	82.8

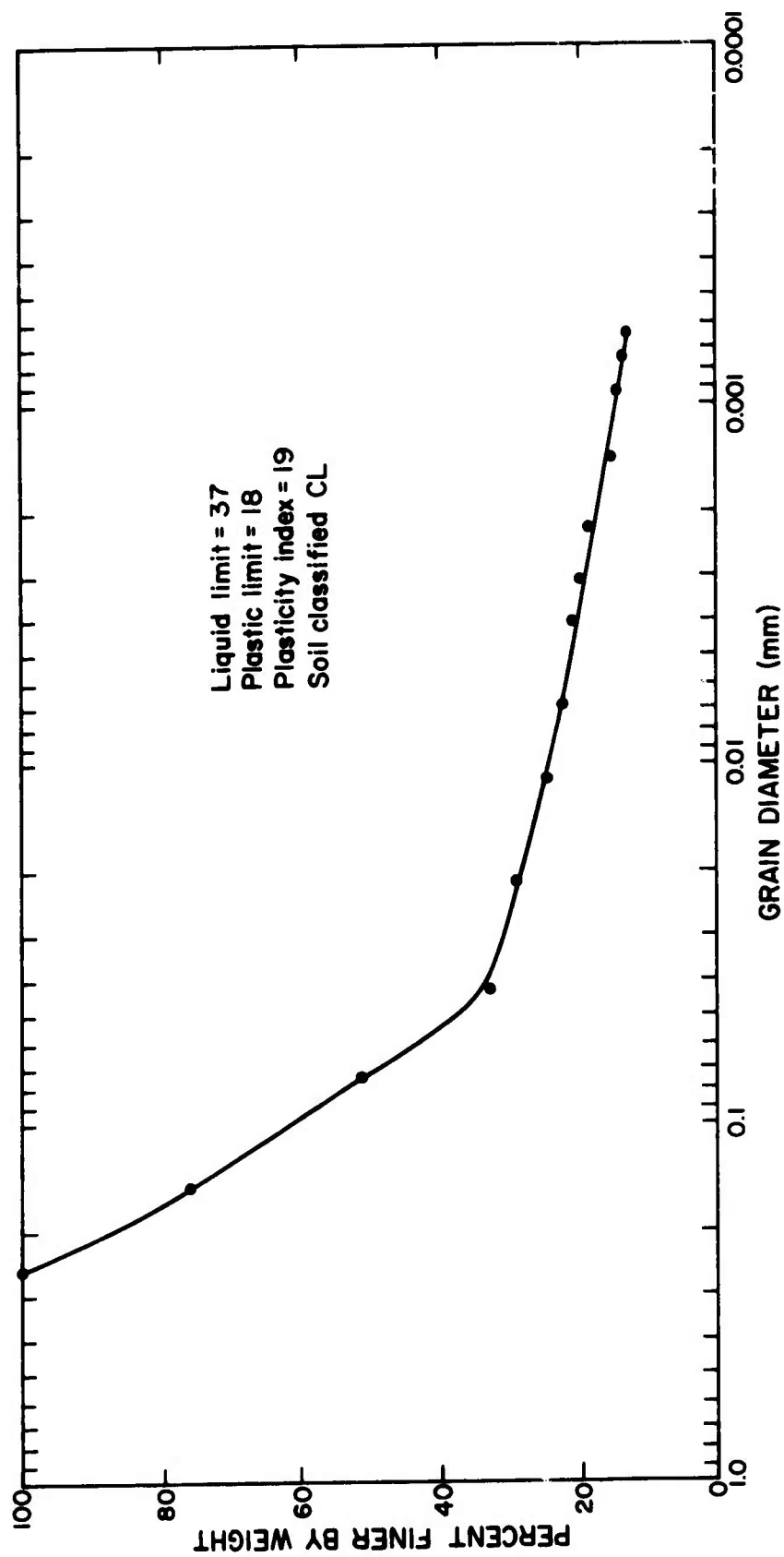


Figure 2. Modified Proctor Compaction of Gravel

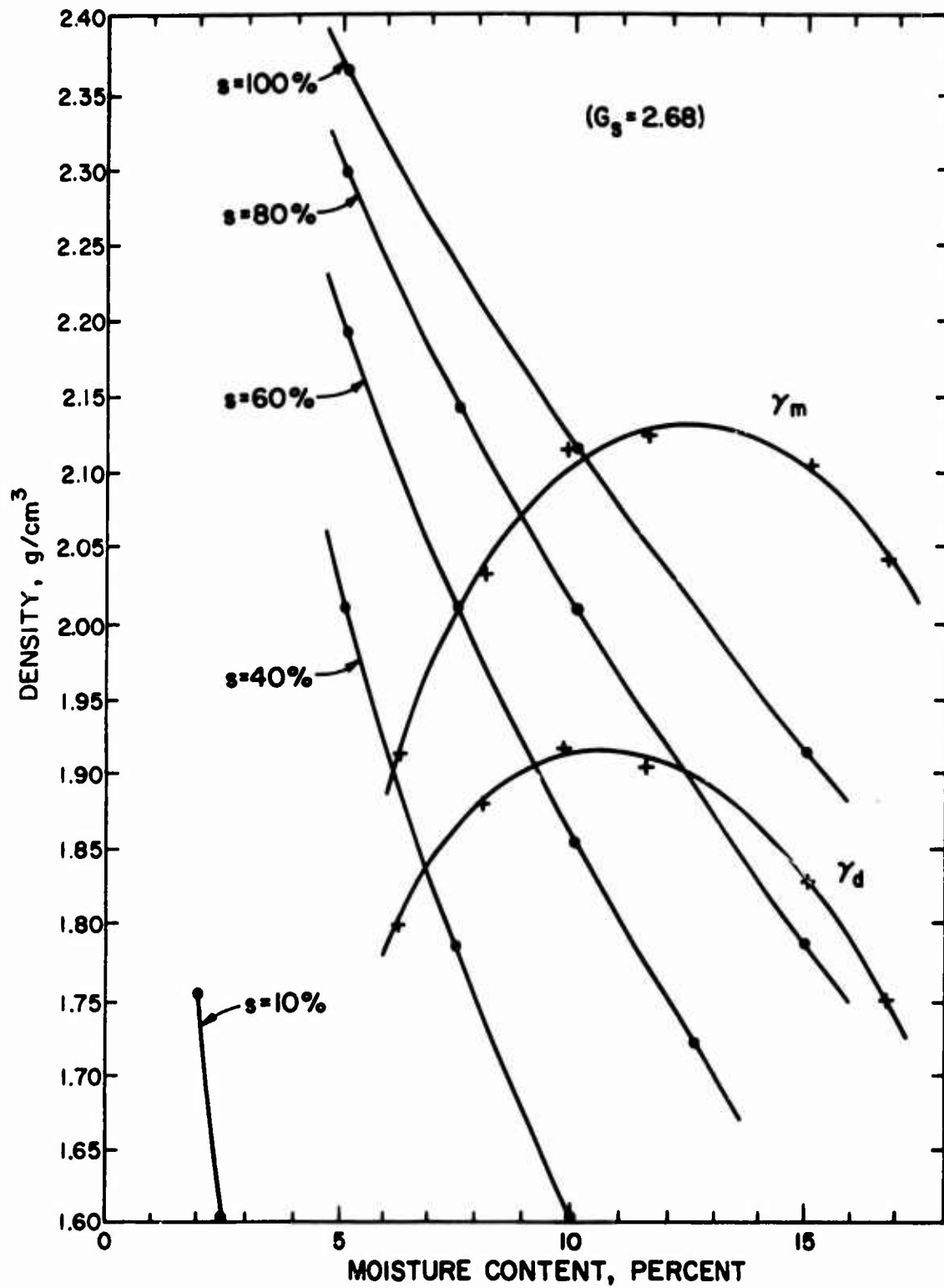


Figure 4. Modified Proctor Compaction of Clay

SECTION III

EXPERIMENTAL RESULTS

Preliminary experiments showed that by sealing the samples in polyethylene the moisture was retained in the sample in a fairly uniform state. A measure of this uniformity was obtained by comparing surface dielectric constant data with the dielectric constant for the layer.

The measurements were made with a commercial microwave network analyzer with an accuracy of ± 0.5 db. This particular instrument would be expected to produce the greatest skin depth uncertainty at the lower frequencies and saturation values. Table 2 contains information on the sample thickness used for testing the various samples at the three different frequencies. The thin samples at high saturation and high frequency were necessitated because of the high attenuation of the signal in these materials.

Table 2
SAMPLE THICKNESS

Soil	Percent Saturation	Frequency (GHz)	Thickness (cm)
Gravel	10	0.5	20.2
		2.5	20.2
		4.5	20.2
	50	0.5	20.2
		2.5	20.2
		4.5	7.6
	90	0.5	20.2
		2.5	20.2
		4.5	9.7
Clay	10	0.5	20.2
		2.5	20.2
		4.5	10.8
	50	0.5	8.9
		2.5	8.9
		4.5	8.9
	90	0.5	2.5
		2.5	2.5
		4.5	2.5

The skin depths calculated from the measured data are shown in table 3. A statistical analysis of this data was performed in an attempt to quantify some preliminary observations. Because the experiment performed was a modified 2^3 factorial design, it produced more data than actually needed for a 2^3 statistical analysis. To make the calculations as simple as possible and still retain data representative of actual field conditions, analysis was performed on two sets of data. The first set of data follows:

Type Soil	Percent Saturation	Frequency (GHz)	Skin Depths	
			Replication 1	Replication 2
Clay	10	0.5	35.8	33.1
		2.5	19.3	13.8
	50	0.5	10.1	12.9
		2.5	8.4	5.6
Gravel	10	0.5	507.2	980.0
		2.5	77.9	73.2
	50	0.5	304.7	196.1
		2.5	19.8	13.3

Treating the soil type as factor A, the degree of saturation as factor B, and the frequency as factor C, an analysis of variance was performed on these data, with the following results:

<u>Analysis of Variance</u>	<u>D.F.</u>	<u>S.S.</u>	<u>M.S.</u>	<u>F</u>
Replications	1	7475.26	7475.26	---
Treatments	7	912827.06	130403.87	8.28*
Error	7	110202.24	15743.18	---

* Significant at the 1 percent level

The individual factor effects were further analyzed as follow:

<u>Factor</u>	<u>S.S.</u>	<u>F</u>
A	258495.98	16.42*
B	85395.45	5.42*
AB	67717.05	4.30
C	213698.18	13.57**
AC	193050.39	12.26*
BC	50254.43	3.19
ABC	44215.58	2.81

* Significant at the 1 percent level

** Barely misses being significant at the 5 percent level

Table 3
MEASURED SKIN DEPTHS

Type Soil	Percent Saturation	Frequency (GHz)	Skin Depths (cm)	
			Replication 1	Replication 2
Clay	10	0.5	35.8	33.1
		2.5	19.3	12.8
		4.5	10.0	11.2
	50	0.5	10.1	12.9
		2.5	8.4	5.6
		4.5	6.3	8.5
	90	0.5	5.1	6.2
		2.5	4.1	3.4
		4.5	2.0	2.0
Gravel	10	0.5	507.2	980.0
		2.5	77.9	73.2
		4.5	25.0	17.3
	50	0.5	304.7	196.1
		2.5	19.8	13.3
		4.5	6.0	4.6
	90	0.5	57.2	44.1
		2.5	5.6	9.0
		4.5	2.5	2.5

As a further check on the validity of the data analysis, a second set of data was selected for a similar 2^3 statistical analysis. The data shown below were used in this set of calculations:

Type Soil	Saturation	Frequency (GHz)	Skin Depths	
			Replication 1	Replication 2
Clay	50	0.5	10.1	12.9
		2.5	8.4	5.6
	90	0.5	5.1	6.2
		2.5	4.1	3.4
Gravel	50	0.5	304.7	196.1
		2.5	19.8	13.3
	90	0.5	57.2	44.1
		2.5	5.6	9.0

Again treating the soil type as factor A, the degree of saturation as factor B, and the frequency as factor C, an analysis of variance was performed on these data, with the following results:

<u>Analysis of Variance</u>	<u>D.F.</u>	<u>S.S</u>	<u>M.S.</u>	<u>F.</u>
Replications	1	967.21	967.21	---
Treatments	7	100523.06	14360.44	19.90*
Error	7	5051.17	721.60	---

* Significant at the 1 percent level

The individual factor effects were analyzed as follows:

<u>Factor</u>	<u>S.S.</u>	<u>F.</u>
A	22052.24	30.56*
B	11891.92	16.48*
AB	9990.00	13.84*
C	20107.24	27.86*
AC	18333.16	25.41*
BC	9321.92	12.92*
ABC	8826.60	12.23**

* Significant at the 1 percent level

** Significant at the 5 percent level

In each test it is quite clear that the soil type is significant at the 1 percent level, thus indicating that the skin depth is highly dependent on the soil type. However, it is also apparent that the moisture content is highly significant, thus indicating that the moisture content of the various layers of the pavement system would have to be known to determine their thicknesses by this method. The frequency of the microwave signal is also a significant factor in the test, and much greater skin depths are measured at the lower frequencies. The interaction effects are also quite significant, especially at the higher moisture contents.

In addition to the above statistical analysis, it is quite apparent from looking at the data in table 3 that there is a huge variation from replication 1 to replication 2 in some of the gravel samples. This amount of variation makes the possibility of accurate determination of layer thickness in gravel very remote using this technique.

SECTION IV

CONCLUSIONS

The objective of this effort was to investigate relationships that exist between microwave penetration, soil type, moisture content, and microwave frequency, and thus determine if it is feasible to use microwave techniques to accurately measure the thickness of various soil layers in a pavement system.

The analysis of data in section III clearly indicates that the moisture content plays a significant role in the attenuation of microwave signals. Therefore, it is necessary to know the moisture content of each layer and its effect on attenuation within that soil before the thickness of that layer can be determined. There is presently no way to nondestructively determine the moisture content of the individual layers, so there appears to be no way to isolate the moisture factor and make the microwave system work.

There are several aspects of equipment selection and usage which could be considered to improve the accuracy of the data collected. Mismatch between antennas and the measuring system could produce a systematic error which could be minimized by using VSWR antennas. The accuracy of the microwave network analyzer could be improved, and additional work could be done to confirm the assumption of single ray theory used in these measurements. It is unlikely, however, that any of these steps could overcome significance of the moisture content on the measurements.

Because of the statistical significance of the moisture content and the interactive effects, and the complexity of isolating these effects, it is recommended that no further research be performed in an attempt to use microwave techniques to measure pavement layer thicknesses.

APPENDIX

THEORETICAL MODEL

The following equations outline the properties of a complex medium (see ref. 1). In general, wave propagating in a medium undergo attenuation and phase shift as they propagate. The complex propagation constant for plane waves is

$$\gamma = j\omega \sqrt{\mu\epsilon} \left[1 - j \frac{\sigma}{\omega\epsilon} \right]^2 \quad (1)$$

and can be written as

$$\gamma = \alpha(\omega) + j\beta(\omega) \quad (2)$$

Solving for $\alpha(\omega)$ and $\beta(\omega)$ gives

$$\alpha(\omega) = \left\{ \frac{\omega^2 \mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right] \right\}^{\frac{1}{2}} \quad (3)$$

$$\beta(\omega) = \left\{ \frac{\omega^2 \mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} + 1 \right] \right\}^{\frac{1}{2}} \quad (4)$$

which for small losses become

$$\alpha(\omega) \approx \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} \quad (5)$$

$$\beta \approx \omega \sqrt{\mu\epsilon} \quad (6)$$

The intrinsic impedance of a medium is given by

$$\eta = \sqrt{\frac{\mu}{\epsilon \left(1 - j \frac{\sigma}{\omega \epsilon} \right)}} \quad (7)$$

The model used consisted of one layer of finite thickness, of the material under test sandwiched between air and an aluminum sheet (see figure 5). The medium is assumed to be isotropic and homogeneous with plane boundaries. Electrical properties of the layer are described by permeability (μ), permittivity (ϵ), and conductivity (σ). The layer is assumed nonmagnetic ($\mu = \mu_0$, where $\mu_0 = 4 (10)^7$ hy/m). Additionally, ϵ is assumed to be independent of frequency. Ray paths for electromagnetic energy are also assumed.

Due to the directivity of the ideal horns assumed in the model, only reflected waves are received. In practice, considerable design placement and polarization strategy are required to reduce the direct coupling to an acceptable level. The horns used were not ideal, however a single ray normal incidence model was used here.

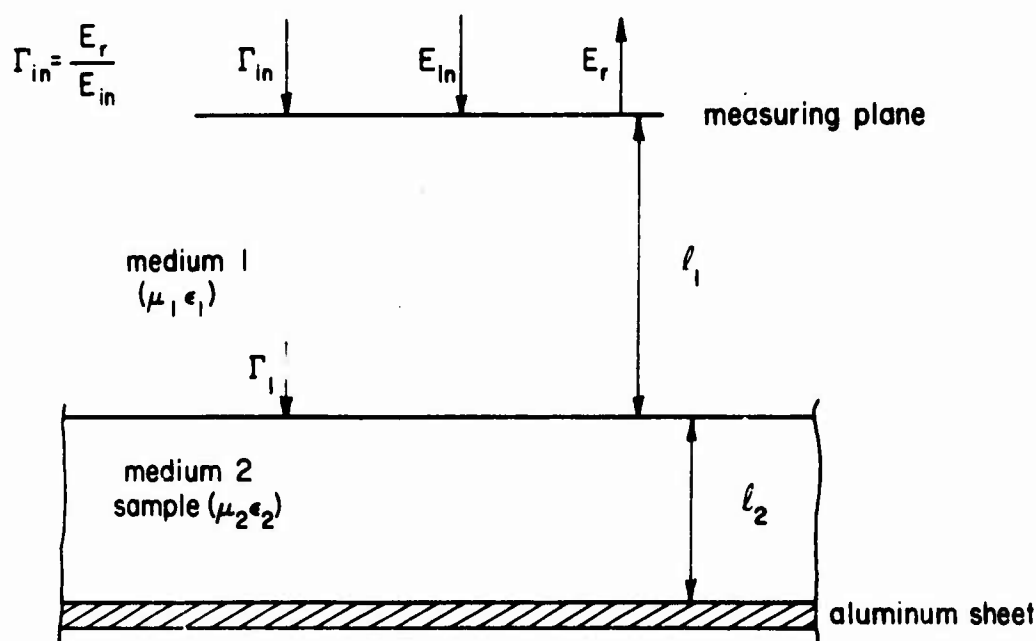


Figure 5. Plane Waves Normally Incident on a Plane Dielectric Boundary

The input reflection coefficient in medium 1 for a single layer can be written as (see ref. 2)

$$\Gamma_{in} = \frac{\Gamma_1 - e^{-2\gamma_2 \ell_2}}{1 - \Gamma_1 e^{-2\gamma_2 \ell_2}} \quad (8)$$

where Γ_1 is the reflection coefficient of a semi-infinite medium and is approximately given by

$$\Gamma_1 = \frac{1 - \sqrt{\epsilon_2'}}{1 + \sqrt{\epsilon_2'}} \quad (9)$$

and γ_2 by (2), (3), and (4). The magnitude of Γ_{in} will go through successive maxima and minima as a function of frequency, layer thickness, dielectric constant and loss. Minima (or maxima) will occur approximately every 2π radians. Thus, when ω is varied, successive minima occur when

$$2(\Delta\beta_2) = 2\pi \quad (10)$$

from which

$$\sqrt{\epsilon_2'} = \frac{c}{2\ell_2 (f_2 - f_1)} \quad (11)$$

where successive minima occur at frequencies f_1 and f_2 .

It has been shown (see ref. 2) that an approximate expression for the maxima of Γ_{in} can be obtained when $\cos 2\beta_2\ell_2 = 1$. Equation (8) then becomes

$$\Gamma_m = \frac{\Gamma_1 - e^{-2\alpha_2\ell_2}}{1 - \Gamma_1 e^{-2\alpha_2\ell_2}} \quad (12)$$

Solving for the attenuator factor

$$-2\alpha_2\ell_2 = \ln \left[\frac{\Gamma_1 - \Gamma_m}{1 - \Gamma_1\Gamma_m} \right] \quad (13)$$

The reflection Γ_1 as given in (9) will always be negative, thus the coefficient Γ_m as given in (12) is also negative. Because $|\Gamma_m| \geq |\Gamma_1|$, logarithms of numbers are involved.

Figure 6 shows how the reflection coefficients in this model are related. The $|\Gamma_1|$ and $|\Gamma_{in}|$ are deduced from measurements. The $|\Gamma_1|$ is determined by using frequencies f_1 and f_2 in (11). The $|\Gamma_{in}|$ is determined by return loss measurement techniques (see ref. 3). The sample thickness is held constant thus the attenuation of the sample can be determined from (13).

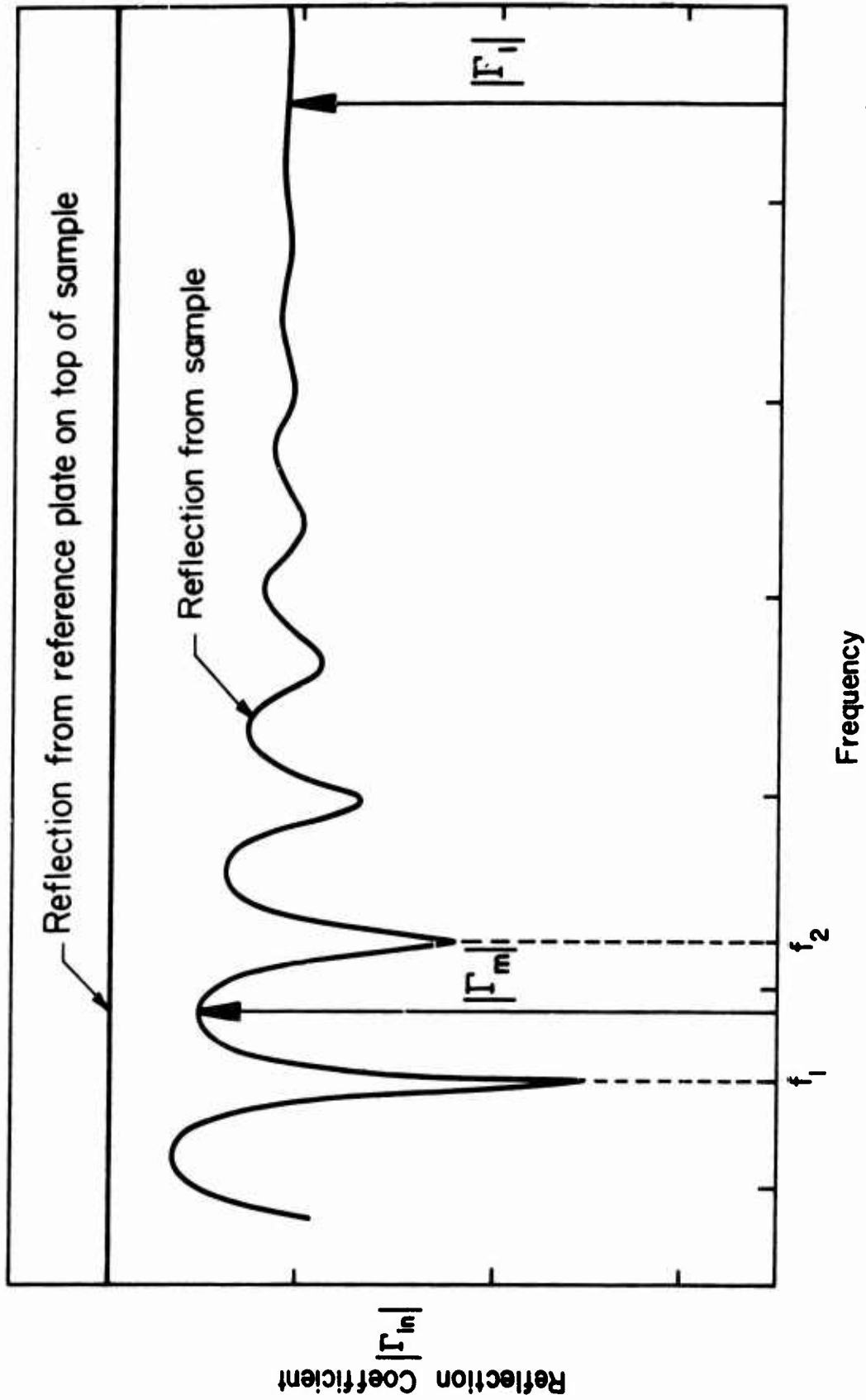


Figure 6. Reflection Coefficient Relationship

Skin depth is defined as the depth at which the wave has been attenuated to $1/e$ of its original value. Thus, skin depth, δ , can be determined from the relation

$$\alpha_2 \delta = 1 \quad (14)$$

ABBREVIATIONS AND SYMBOLS

dBm	Signal level with respect to 1 mW, decibels
E_{in}	Incident electric intensity, v/m
E_r	Reflected electric intensity, v/m
G_r	Grain specific gravity
S	Degree of saturation
c	Velocity of light in free space, m/sec
e	Natural logarithm base
f	Frequency, Hz
i	1, 2 for media 1 and 2 respectively
l_2	Sample depth, m
t	Time, sec.
Γ	Reflection coefficient of infinitely thick sample
Γ_{in}	Reflection coefficient of finite thickness sample
Γ_m	Maximum reflection coefficient of finite thickness sample
α_i	Attenuation constant, nepers/m
β_i	Phase constant, radians/m
γ_i	Propagation constant
γ_d	Dry density, g/cm
γ_m	Wet density, g/cm
δ	Skin depth, m
ϵ_i	Permittivity, farad/m
ϵ'_i	Permittivity relative to free space, farad/m
μ_i	Permeability, henry/m
σ	Conductivity, mho/m
ω	Angular velocity, radians/sec
η	Intrinsic impedance, ohm

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